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Emergent radiation in an atom–field system at twice resonance

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Abstract

A two-level atom interacting with a single mode of quantized electromagnetic radiation is discussed using a new representation in which the atom and the radiation are unified into a *single* canonical Bose field. At the *twice resonance*, when the frequency of the original radiation is twice the atomic transition frequency ($\omega = 2\epsilon$), the *emergent* unified field in the non-interacting atom–field system resembles a free radiation of frequency ϵ. This free emergent radiation is further shown to exist in the presence of an interaction which looks similar to the atom–field interaction in the dipole approximation. The one-photon correlation and the population inversion are discussed as the possible means of observing the emergent radiation. The entanglement properties of the emergent radiation are also discussed.

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1. Introduction

Matter–radiation interaction problems are of central importance to all branches of physics. A typical problem of this kind, in quantum optics for example, concerns the interaction of light with an atom in the long wavelength (dipole) approximation [1, 2]. In a useful simplification of the atom–field problem, the atom is often approximated as an effective two-level system. In a fully quantum-theoretic formulation, the light is treated as a quantized radiation field. The corresponding atom–field Hamiltonians, for example the Dicke maser model [3] or the Rabi model [4], form the basis of understanding for a number of physical phenomena, such as the cooperative superradiant emission [3], or the oscillations of the population inversion and their collapse and revival [5–7].

In this paper, we consider a two-level atom interacting with a single mode of quantized radiation and predict the emergence of a *free* unified radiation at the twice resonance. The term twice resonance refers to the condition when the frequency of the original radiation is twice the atomic transition frequency. The present study is based on, and inspired by, a representation in which a two-level atom and a quantized single-mode radiation are unified into a *single* Bose field [8]. The existence of this free emergent radiation, whose frequency is the same as the atomic transition frequency, is first demonstrated in the non-interacting case. It is then shown to exist in an interacting system, which is similar to the atom–photon model in the dipole approximation. We discuss the time dependence of the one-photon correlation and the population inversion in simple situations to illustrate its physical effects. We also study the atom–photon entanglement in an emergent radiation state.

Let us first very briefly recall the two-level atom and the quantized radiation. Through this, we also fix the notations and vocabulary for our main discussion. Let $|g\rangle$ and $|e\rangle$ denote the ground and excited states of a two-level atom, respectively. Let ϵ be the energy difference of the two atomic levels and $\bar{\epsilon}$ be their average. The Hamiltonian of such an isolated two-level atom can be written as $H_{\text{atom}} = \bar{\epsilon}\mathbb{I} + \frac{\epsilon}{2}\sigma^z$, where $\mathbb{I}$ is the identity operator and $\sigma^z = |e\rangle\langle e| - |g\rangle\langle g|$ is the z-component Pauli operator. Physically, σ^z measures the population inversion (the difference of probabilities of finding the atom in the excited and ground states). The transition between the atomic levels is described in terms of the other two Pauli operators, $\sigma^+ = |e\rangle\langle g|$ and $\sigma^- = |g\rangle\langle e|$. Together, σ^z and $\sigma^\pm$ satisfy the usual spin-1/2 operator algebra.

A single mode of the quantized electromagnetic radiation is described in terms of the Bose operators, $\hat{b}$ and $\hat{b}^\dagger$, which annihilate and create a photon, respectively. These operators act in the Fock space of photons, spanned by the basis, $\{|m\rangle, m = 0, 1, \ldots, \infty\}$, such that $\hat{b}|m\rangle = \sqrt{m}|m-1\rangle$ and $\hat{b}^\dagger|m\rangle = \sqrt{m+1}|m+1\rangle$. The ket $|m\rangle$ denotes the m-photon state for the given mode of radiation. The Hamiltonian of such a single-mode quantized radiation can be written as $H_{\text{field}} = \omega\big(\hat{b}^\dagger\hat{b} + \frac{1}{2}\big)$, where $\hat{b}^\dagger\hat{b}$ is the photon number operator, that is, $\hat{b}^\dagger\hat{b}|m\rangle = m|m\rangle$ and ω is the photon energy ($\hbar = 1$).

The interaction of a two-level atom with a single mode of quantized radiation is given, in the dipole approximation, by $V_{\text{dipole}} = (\hat{b}^\dagger + \hat{b})\sigma^x$, where $\sigma^x = \sigma^+ + \sigma^-$ measures the electric dipole of the atom and the electric field of the radiation is proportional to $\hat{b}^\dagger + \hat{b}$. The simplest analysis of the matter–radiation problem can therefore be carried out in terms of the Hamiltonian, $H_{\text{field}} + H_{\text{atom}} + gV_{\text{dipole}}$, where g is the dipole–radiation coupling [1]. This model has been of fundamental interest to studies on quantum optics and magnetic resonance [4]. In the rotating wave approximation due to Jaynes and Cummings, for ω close to ϵ, V_{dipole} is approximated by $V_{JC} = (\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+)$, by ignoring the faster processes, $\hat{b}^\dagger\sigma^+ + \hat{b}\sigma^-$. This simplification results in the exactly solvable Jaynes–Cummings (JC) model [9]. The interaction arising in the present study is similar to V_{dipole}, but not the same.

The remainder of the paper is organized as follows. In the following section, first we present the unified boson representation which is the nucleus of this work. Then, we introduce the idea of an emergent radiation, through an observation in the non-interacting atom–field system. In section 3, we discuss the existence of the emergent radiation in an exactly solvable interacting model. We discuss the one-photon correlation and population inversion in section 4. There, we try to understand the physical observable effects of the emergent radiation. We also discuss the atom–photon entanglement in section 5. Finally, we conclude with a summary and some observations.

2. Unified boson representation

Recently, we have invented a new representation in which the radiation operators, $\hat{b}$ and $\hat{b}^\dagger$, and the atomic operators, σ^z and $\sigma^\pm$, are unified into a single canonical Bose operator [8]. Let $\hat{a}^\dagger$ be the creation operator of the unified boson. Then, according to our representation,

$$\hat{a}^\dagger = \sqrt{2}\Big[\sqrt{\hat{b}^\dagger\hat{b} + \tfrac{1}{2}}\,\sigma^+ + \hat{b}^\dagger\sigma^-\Big]. \tag{1}$$

The operators, $\hat{a}^\dagger$ and $\hat{a}$, satisfy the bosonic commutation relations and act on the new Fock states, $\{|n\rangle, n = 0, 1, \ldots, \infty\}$, defined as $|n = 2m\rangle = |m\rangle \otimes |g\rangle$ and $|n = 2m+1\rangle = |m\rangle \otimes |e\rangle$. The representation in equation (1) is based on this definition of the $|n\rangle$ kets (please refer to appendix B of [8] for details). The corresponding inverse representation is given by the equations:

$$\sigma^z = -\cos(\pi \hat{a}^\dagger \hat{a}) := -\hat{\chi}, \tag{2}$$

$$\sigma^+ = \frac{1-\hat{\chi}}{2} \frac{1}{\sqrt{\hat{N}}} \hat{a}^\dagger \tag{3}$$

and

$$\hat{b}^\dagger = \frac{\hat{a}^\dagger \hat{a}^\dagger}{\sqrt{2}} \left(\frac{1-\hat{\chi}}{2} \frac{1}{\sqrt{\hat{N}+2}} + \frac{1+\hat{\chi}}{2} \frac{1}{\sqrt{\hat{N}+1}} \right), \tag{4}$$

where $\hat{\chi}$ can also be written as $(-)^{\hat{N}}$ and $\hat{N} = \hat{a}^\dagger \hat{a}$ is the number operator of the unified boson. The inverse representation is completely consistent, as it satisfies the necessary algebra[1].

2.1. Emergent radiation at twice resonance

Through the inverse transformation, we can convert any problem of a two-level atom interacting with a single-mode of quantized radiation to an equivalent problem of the unified boson field. In general, it will be a highly nonlinear unified field problem. The corresponding energy eigenstates are not expected to be like that of a free electromagnetic radiation (the hallmark of which is the equidistant successive photon states like a simple harmonic oscillator). The unified field in an arbitrary atom–field system will therefore be different from a normal free radiation. However, below we present a simple situation, in which the emergent unified field completely resembles a free radiation. We will refer to this 'free' unified field as the emergent radiation.

Consider the free-field Hamiltonian for the unified radiation. Up to an overall factor of energy, it is

$$\hat{a}^\dagger \hat{a} + \tfrac{1}{2} = 2\left(\hat{b}^\dagger \hat{b} + \tfrac{1}{2}\right) + \tfrac{1}{2}\sigma^z. \tag{5}$$

The right-hand side of the above equation shows the corresponding Hamiltonian in terms of the original atom–field variables. It just happens to be a non-interacting atom–radiation problem at precisely the twice resonance. It also implies that the 'photon' energy of the emergent radiation is the same as the atomic transition frequency. Thus, equation (5) presents a special atom–field system, in which the unified radiation emerges as 'free'. With the benefit of hindsight, now we can directly demonstrate the existence of this free emergent radiation without evoking the representation. In figure 1, we present how, at twice resonance, the energy spectrum of the non-interacting atom–photon system is *indistinguishable* from that of a free radiation with photon energy ϵ. Demanding that the successive eigenstates be equally spaced leads to the condition, $\omega = 2\epsilon$.[2]

In experimental terms, it suggests that if there is a two-level atom inside a cavity with precisely twice-resonant field, then the radiation effects of this system will, in principle, be

[1] We have also derived the unified boson representation for the three-level atom (discussed elsewhere). In principle, it can be done for the multi-level atoms also.

[2] One also finds equally spaced eigenstates in the non-interacting atom–photon system for $\epsilon = l\omega$, where l is a positive integer. In this case, however, the lowest l states are non-degenerate, while the rest are two-fold degenerate. It is therefore *different* from a free radiation. Moreover, the level spacing in this case is ω, not ϵ.

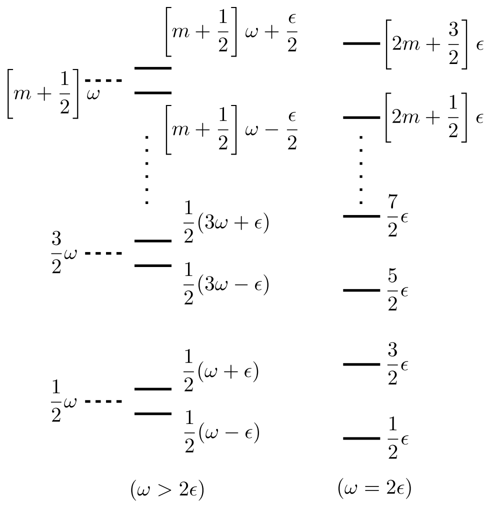

Figure 1. Energy-level diagram of a quantized single-mode radiation and a two-level atom in the non-interacting case. For $\omega = 2\epsilon$, the states of the atom–field system are indistinguishable from that of a radiation alone.

indistinguishable from that of another cavity with only the radiation of the frequency ϵ. For example, a test atom, with transition frequency close to ϵ, will undergo Rabi oscillations inside such a cavity. Or, a cavity with the forbidden ϵ-frequency mode (but allowed 2ϵ modes) will actually exhibit or sustain an ϵ-frequency radiation in the presence of a right atom. The emergence of a free unified radiation in the non-interacting system, however, presents a trivial case. Therefore, it is important to ask whether this emergent radiation will survive in an interacting atom–photon system, or not[3].

3. Emergent radiation in an interacting atom–photon model

In order to investigate the emergent radiation in the presence of an atom–photon interaction, we introduce such modifications in the free-field Hamiltonian of the unified radiation (equation (5)) that the equidistant character of its eigenstates survives. Clearly, the following general Hamiltonian fulfils our objective,

$$H = \epsilon\left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right) + \xi(\hat{a}^\dagger + \hat{a}) + \eta(\hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}). \tag{6}$$

It is an exactly solvable model with a displacement term ($\propto \xi$) and a squeezing ($\propto \eta$). This H is diagonalized by the unitary transformations, $\mathcal{D}(x) = \mathrm{e}^{-x(\hat{a}^\dagger - \hat{a})}$ and $\mathcal{U}(\theta) = \mathrm{e}^{-\theta(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a})}$, where $x = \frac{\xi}{\epsilon+2\eta}$ and $\theta = \frac{1}{4}\tanh^{-1}\left(\frac{2\eta}{\epsilon}\right)$. For every ket $|n\rangle$ of the a-boson, the exact eigenstate of H is given by $|\psi_n\rangle = \mathcal{D}(x)\mathcal{U}(\theta)|n\rangle$, with an eigenvalue, $E_n = \sqrt{\epsilon^2 - 4\eta^2}\left(n + \frac{1}{2}\right) - \frac{\xi^2}{(\epsilon+2\eta)}$. While ξ uniformly lowers the eigenvalue for each n, η renormalizes the frequency of the unified radiation. However, the basic structure of the eigen spectrum of H remains like that of a free radiation (that is, $E_n \propto (n + \frac{1}{2})$). The emergent radiation eigenstate, $|\psi_n\rangle$, is now an *entangled* state of the atom and the original radiation, unlike in the non-interacting case, where it is not entangled.

[3] It is encouraging to note that the free-field behavior of the emergent radiation definitely survives to the first order in coupling g in H_{dipole}, because the leading corrections to the energy eigenvalues of H_{dipole} appear in $\mathcal{O}(g^2)$.

By using equation (1), this H can be converted to the following atom–field problem:

$$H = 2\epsilon H_b + \frac{\epsilon}{2}\sigma^z + \xi\sqrt{2}(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+ + \sqrt{H_b}\sigma^x) + \eta\{\sqrt{H_b}[(\hat{b}^\dagger + \hat{b}) + \sigma^z(\hat{b}^\dagger - \hat{b})] + \text{h.c.}\}, \tag{7}$$

where $H_b = \hat{b}^\dagger\hat{b} + \frac{1}{2}$. According to this form, the atom and the radiation interact via different terms arising due to ξ and η. The atom–field interactions in equation (7) look rather complicated. However, note the interaction, $\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+$, arising due to ξ. It is the well-known Jaynes–Cummings interaction, V_{JC}. To keep things simple, we therefore set $\eta = 0$ for the rest of our discussion. A non-zero ξ is sufficient to generate the desired atom–photon interaction.

We now have a sufficiently non-trivial but a simple model in H for $\eta = 0$, which preserves the free-field character of the emergent radiation at twice resonance. The corresponding atom–photon Hamiltonian is written as

$$H_0 = 2\epsilon H_b + \frac{\epsilon}{2}\sigma^z + \xi\sqrt{2}(\hat{b}^\dagger\sigma^- + \hat{b}\sigma^+ + \sqrt{H_b}\sigma^x). \tag{8}$$

Besides the physical V_{JC}, H_0 also has another piece to the interaction, that is $\sqrt{H_b}\sigma^x$, which has not been encountered before. This new interaction cannot be ignored within the rotating wave approximation, as its time dependence (in the interaction picture) is similar to that of V_{JC}. While $\sqrt{H_b}\sigma^x$ evolves as $\sqrt{H_b}(\sigma^+\mathrm{e}^{\mathrm{i}\epsilon t} + \sigma^-\mathrm{e}^{-\mathrm{i}\epsilon t})$, V_{JC} varies as $\hat{b}^\dagger\sigma^-\mathrm{e}^{\mathrm{i}\epsilon t} + \hat{b}\sigma^+\mathrm{e}^{-\mathrm{i}\epsilon t}$. Clearly, we cannot drop either of the two on the grounds of being faster. Moreover, we need both of these anyway for maintaining the free-field character of the emergent radiation. Therefore, it would be nice to have some physical understanding for $\sqrt{H_b}\sigma^x$, so that it can at least formally be considered realistic. Surprisingly, we can show that $\sqrt{H_b}\sigma^x$ is a unitary transformed version of the interaction, $\hat{b}^\dagger\sigma^+ + \hat{b}\sigma^-$ (which gets dropped from V_{dipole} in the rotating wave approximation).

To establish this unitary connection, consider the original radiation in the number–phase representation. That is, $\hat{b}^\dagger = \sqrt{\hat{M}}\mathrm{e}^{-\mathrm{i}\hat{\Phi}}$ and $\hat{b} = \mathrm{e}^{\mathrm{i}\hat{\Phi}}\sqrt{\hat{M}}$, where the Hermitian operators $\hat{M}$ and $\hat{\Phi}$ are the number and phase operators, respectively[4]. Clearly, $\hat{M} = \hat{b}^\dagger\hat{b}$ and $\mathrm{e}^{\mathrm{i}\hat{\Phi}}\hat{M}\,\mathrm{e}^{-\mathrm{i}\hat{\Phi}} = \hat{M} + 1$. The latter also implies $[\hat{M}, \hat{\Phi}] = \mathrm{i}$. In this representation, $\hat{b}^\dagger\sigma^+ + \hat{b}\sigma^- = \sqrt{\hat{M}}\,\mathrm{e}^{-\mathrm{i}\hat{\Phi}}\sigma^+ + \sigma^-\mathrm{e}^{\mathrm{i}\hat{\Phi}}\sqrt{\hat{M}}$. It prompts us to absorb the phase operator into $\sigma^\pm$. To achieve this, we introduce a unitary transformation, $\mathcal{U}_{\hat{\Phi}} = \mathrm{e}^{-\frac{\mathrm{i}}{2}\sigma^z\hat{\Phi}}$, such that $\mathcal{U}_{\hat{\Phi}}^\dagger\sigma^\pm\mathcal{U}_{\hat{\Phi}} = \sigma^\pm\,\mathrm{e}^{\pm\mathrm{i}\hat{\Phi}}$. Interestingly, under this transformation, we get

$$\mathcal{U}_{\hat{\Phi}}^\dagger(\hat{b}^\dagger\sigma^+ + \hat{b}\sigma^-)\mathcal{U}_{\hat{\Phi}} = \sqrt{\hat{M} + \tfrac{1}{2}}\sigma^x = \sqrt{H_b}\sigma^x. \tag{9}$$

This is a beautiful result. It offers a meaningful comparison between the interaction in H_0 and V_{dipole}. It also suggests that, to physically realize the emergent radiation state, one may need to (selectively) slow down the fast processes of V_{dipole}, without affecting the slow terms.

4. Physical effects

Now we discuss some physical results within H_0, concerning the emergent radiation.

[4] Although the number–phase representation has a limitation due to $|m = 0\rangle$ state, we use it in the spirit of a coherent state, as the emergent radiation eigenstates involve a sum over all $|m\rangle$.

4.1. One-photon correlation: sub-harmonic effect

We first compute the time-dependent correlation function of the electric field, $E_b(t)$, of the original radiation. Since $E_b(t) \propto \hat{b}^\dagger(t) + \hat{b}(t)$, the corresponding one-photon correlation $\langle E_b(t)E_b(0)\rangle$ is proportional to $\mathcal{G}(t)$, where

$$\mathcal{G}(t) = \langle[\hat{b}^\dagger(t) + \hat{b}(t)][\hat{b}^\dagger(0) + \hat{b}(0)]\rangle. \tag{10}$$

Here, $\hat{b}(t) = \mathrm{e}^{\mathrm{i}H_0 t}\hat{b}\,\mathrm{e}^{-\mathrm{i}H_0 t}$, and for any operator $\hat{O}$, the expectation $\langle\hat{O}\rangle = \mathrm{tr}\{\hat{\rho}\hat{O}\}$, where $\hat{\rho}$ is a density operator. Since $\mathcal{D}^\dagger(x)H_0\mathcal{D}(x) = \tilde{H}_0 = \epsilon\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) - \frac{\xi^2}{\epsilon}$ for $x = \xi/\epsilon$, we also transform $\hat{b}^\dagger$ to $\hat{b}^\dagger(x) = \mathcal{D}^\dagger(x)\hat{b}^\dagger\mathcal{D}(x)$, and similarly $\hat{\rho}$ to $\hat{\rho}(x)$. Therefore, $\mathcal{G}(t) = \mathrm{tr}\{\hat{\rho}(x)[\hat{b}^\dagger(x,t) + \hat{b}(x,t)][\hat{b}^\dagger(x,0) + \hat{b}(x,0)]\}$, where $\hat{b}(x,t) = \mathrm{e}^{\mathrm{i}\tilde{H}_0 t}\hat{b}(x)\,\mathrm{e}^{-\mathrm{i}\tilde{H}_0 t}$. In order to demonstrate how the emergent radiation will manifest itself through $\mathcal{G}(t)$, we discuss two limiting cases: the weak ($x \ll 1$) and the strong coupling ($x \gg 1$). For simplicity, we may take $\hat{\rho}$ to be either the equilibrium density matrix, $\mathrm{e}^{-\beta H_0}/\mathrm{tr}\{\mathrm{e}^{-\beta H_0}\}$, or a pure eigenstate of H_0, that is $|\psi_n\rangle\langle\psi_n|$. For these two choices, $\hat{\rho}(x) = \mathrm{e}^{-\beta\tilde{H}_0}/\mathrm{tr}\{\mathrm{e}^{-\beta\tilde{H}_0}\}$ and $|n\rangle\langle n|$, respectively.

In the weak-coupling limit, we write $\hat{b}^\dagger(x) + \hat{b}(x)$ as

$$\hat{b}^\dagger(x) + \hat{b}(x) \approx (\hat{b}^\dagger + \hat{b}) - x\sqrt{2}\{(\sigma^+ + \sigma^-) - [\sqrt{H_b}, \hat{b}^\dagger + \hat{b}](\sigma^+ - \sigma^-)\}, \tag{11}$$

where the terms of order $\mathcal{O}(x^2)$ have been ignored. In this limit, we get the following expression for $\mathcal{G}(t)$:

$$\begin{aligned}\mathcal{G}(t)_{\xi\ll\epsilon} \approx{}& \langle\hat{b}^\dagger\hat{b}\rangle\,\mathrm{e}^{\mathrm{i}2\epsilon t} + \langle\hat{b}\hat{b}^\dagger\rangle\,\mathrm{e}^{-\mathrm{i}2\epsilon t} + 2x^2\{\mathrm{e}^{\mathrm{i}\epsilon t}[\langle\sigma^+\sigma^-\rangle + B_1\langle\sigma^-\sigma^+\rangle]\\ &+ \mathrm{e}^{-\mathrm{i}\epsilon t}[\langle\sigma^-\sigma^+\rangle + B_2\langle\sigma^+\sigma^-\rangle] + B_1\langle\sigma^+\sigma^-\rangle\,\mathrm{e}^{\mathrm{i}3\epsilon t} + B_2\langle\sigma^-\sigma^+\rangle\,\mathrm{e}^{-\mathrm{i}3\epsilon t}\},\end{aligned} \tag{12}$$

where $B_1 = \langle[\sqrt{H_b}, \hat{b}^\dagger][\hat{b}, \sqrt{H_b}]\rangle$ and $B_2 = \langle[\sqrt{H_b}, \hat{b}][\hat{b}^\dagger, \sqrt{H_b}]\rangle$. The presence of the $\mathrm{e}^{\pm\mathrm{i}\epsilon t}$ terms in equation (12) clearly indicates the dynamical generation of the free emergent radiation in the system. It is fascinating that, in a system with the radiation of frequency 2ϵ, a *fundamental* note of frequency ϵ appears due to the interaction. It is as if the original photon has split into two new photons with half the energy. Even for an arbitrary value of x, $\mathcal{G}(t)$ will only have the terms of frequency ϵ and its higher harmonics.

In the strong-coupling limit, we can write

$$\hat{b}^\dagger(x) + \hat{b}(x) \approx \frac{1}{\sqrt{2}}\left[2x - (\hat{a}^\dagger + \hat{a}) + \frac{3}{4x}(\hat{a}^\dagger - \hat{a})^2\right], \tag{13}$$

where the terms of $\mathcal{O}(1/x^2)$ have been ignored. The terms containing $\hat{\chi}(x)$, which roughly fall as e^{-2x^2}/x for large x, have been completely ignored. In this case, the correlation function can be written as

$$\mathcal{G}(t)_{\xi\gg\epsilon} \approx c_0 + \frac{1}{2}[\langle\hat{a}\hat{a}^\dagger\rangle\mathrm{e}^{-\mathrm{i}\epsilon t} + \langle\hat{a}^\dagger\hat{a}\rangle\,\mathrm{e}^{\mathrm{i}\epsilon t}] + \frac{9}{32x^2}[\langle\hat{a}\hat{a}\hat{a}^\dagger\hat{a}^\dagger\rangle\,\mathrm{e}^{-\mathrm{i}2\epsilon t} + \langle\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}\rangle\,\mathrm{e}^{\mathrm{i}2\epsilon t}], \tag{14}$$

where $c_0 = 2x^2 - 3\left[\langle\hat{a}^\dagger\hat{a}\rangle + \frac{1}{2}\right] + \cdots$, is a constant term. The terms, $\mathrm{e}^{\pm\mathrm{i}\epsilon t}$, corresponding to the free emergent radiation, again appear in the correlation function. In fact, a striking feature of equation (14) is that the $\mathrm{e}^{\pm\mathrm{i}2\epsilon t}$ terms of the original radiation are sub-leading (in powers of $1/x$) compared to the emergent radiation terms. Therefore, the strong-coupling case of H_0 presents an *inverse* of the second-harmonic generation effect. This new effect may be called as the half- or *sub-harmonic* generation. Although this effect resembles the parametric down-conversion (PDC) in nonlinear optics [10–12], the two are microscopically different. While the PDC happens due to the interaction between the photons of at least two different modes of the radiation, the (sub-harmonic) emergent radiation is a consequence of the atom–photon 'unification'. For example, the PDC of a photon of energy 2ϵ to ϵ-frequency photons would occur via the scattering process: $\hat{\alpha}^\dagger\hat{\alpha}^\dagger\hat{\beta}$, where $\hat{\alpha}$ is the annihilation operator of the ϵ energy photon and $\hat{\beta}$ is that of the incident (2ϵ energy) photon [13]. In contrast, there is no explicit ϵ-frequency mode of the original radiation present in our problem. Instead, the system collectively behaves as such[5].

4.2. Population inversion: atomic coherent state

Now, we briefly discuss the time evolution of the population inversion. For simplicity, consider only the following choices for the initial state: $|n = 0\rangle = |m = 0\rangle \otimes |g\rangle$ and $|n = 1\rangle = |m = 0\rangle \otimes |e\rangle$. In both cases, the original radiation is in the vacuum state. The exact population inversion in the two cases is as follows: $W_0(t) = \langle 0|\sigma^z(t)|0\rangle = -\mathrm{e}^{-4x^2(1-\cos\epsilon t)}$, and similarly, $W_1(t) = [1 - 8x^2(1 - \cos\epsilon t)]\,\mathrm{e}^{-4x^2(1-\cos\epsilon t)}$. Both W_0 and W_1 oscillate in time with frequency ϵ. Notably, the oscillation frequency is independent of the atom–photon coupling, unlike in the Rabi oscillations. For $x \ll 1$, both W_0 and W_1 make simple sinusoidal oscillations. The behavior for $x \gg 1$ changes to periodic pulsing: $W_0 = -\delta_{t=t_l} = -W_1$, where $t_l = \frac{2\pi}{\epsilon} l$ and l is an integer. That is, both W_0 and W_1 remain zero except at t_l when the atom only momentarily recovers (and loses) the initial state. The above features in a population inversion measurement could be taken as the signatures of an emergent radiation. The zero population inversion implies an atomic coherent state, with an equal probability for the atom to be in $|g\rangle$ and $|e\rangle$. This result offers a method for preparing the atomic coherent states by means of realizing the emergent radiation. It may be of interest to the coherent atom optics.

5. Entanglement in the emergent radiation

We end this paper with a discussion on the atom–photon entanglement in an emergent radiation state. We use a simple measure of bipartite entanglement in a pure state: $\mathcal{E} = 1 - \mathrm{tr}_A\hat{\rho}_A^2 = 1 - \mathrm{tr}_R\hat{\rho}_R^2$, where $\hat{\rho}_A = \mathrm{tr}_R\hat{\rho}$ and $\hat{\rho}_R = \mathrm{tr}_A\hat{\rho}$ are the reduced density operators of the atom and the original radiation, respectively (with $\hat{\rho}$ being the full density operator (a projection operator in a pure state)). Clearly, $\mathcal{E} = 0$ is a necessary and sufficient condition for the separability of a pure state. It is because $\hat{\rho}_A$ and $\hat{\rho}_R$ are also projection operators in such a state (key feature of a separable pure state). Hence, we define the deviation of the reduced density operators from *idempotency*, that is $\mathcal{E}$, as a measure of the entanglement[6].

Applying this measure to the simplest emergent radiation state $|\psi_0\rangle = \mathrm{e}^{-x(\hat{a}^\dagger - \hat{a})}|0\rangle$, which can be written as $|\psi_0\rangle = \mathrm{e}^{-\frac{x^2}{2}} \sum_{m=0}^{\infty} \frac{x^{2m}}{\sqrt{(2m)!}} |m\rangle \otimes \left[|g\rangle - \frac{x}{\sqrt{2m+1}}|e\rangle\right]$, we get

$$\mathcal{E}(x) = \tfrac{1}{2}(1 - \mathrm{e}^{-4x^2}) - 2\mathcal{A}^2(x)\,\mathrm{e}^{-2x^2}, \tag{15}$$

where $\mathcal{A}(x) = x\sum_{m=0}^{\infty} \frac{x^{4m}}{(2m)!}\frac{1}{\sqrt{2m+1}}$. As expected, the entanglement is zero for $x = 0$. For weak couplings, $\mathcal{E}$ increases with x. However, it begins to weaken for $x \gtrsim 1.5$ (see figure 2). In fact, $\mathcal{E}$ tends to 0 for $x \to \infty$, because in this limit, $\mathcal{A}(x) \to \frac{1}{2}\mathrm{e}^{x^2}$. This 'asymptotic' disentanglement of the radiation from the atom is a novel cooperative effect, in which the atom lives in a coherent state, $\frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$ (also noted in the population inversion), supported by an effectively decoupled radiation. Although, for simplicity, we have calculated

[5] It is important to realize that our unified boson is not *independent* of the original radiation. That is, $\hat{a}$ and $\hat{b}$ do not commute. In other words, one either works with $\hat{b}$ and $\vec{\sigma}$, or with $\hat{a}$, not with both simultaneously. In comparison to this, the photon operators $\hat{\alpha}$ and $\hat{\beta}$ of the PDC problem commute, as they correspond to the independent modes of radiation.

[6] Such a measure of entanglement (that is, the linearized entropy of the reduced density operators) is also known in the literature as *tangle* [14, 15].

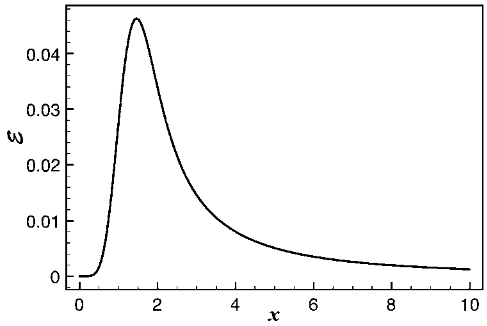

Figure 2. Entanglement, $\mathcal{E}$, versus atom–field coupling, $x = \xi/\epsilon$, in the emergent radiation state $|\psi_0\rangle$.

the entanglement only in $|\psi_0\rangle$, the same calculations can also be done for the higher emergent radiation states, $|\psi_n\rangle$.

6. Conclusion

We have presented the idea of an emergent radiation, clearly stating its novel mathematical basis supported by physical calculations. Although it is discussed in the context of quantum optics, the idea is equally applicable to a magnetic-resonance system or any spin-boson problem, subjected to the corresponding physical interpretations. From a (very contemporary) perspective, an emergent radiation state with its atom–photon entanglement could be of interest to the quantum information community, as it integrates the information carrying 'qubit' with the carrier 'light'. It may be fruitful to investigate such an emergent 'qubit–light' state as a quantum informatic system.

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